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Cash Augmented for Trade: Circulation Currency with Measurement Functionality

Introduction

Monetary value, currency, issuing authority, and year of minting need to be depicted on cash. The coin rim is often marked *decus et tutamen* with stripes and text to discourage clipping and as a usability feature for the blind. Otherwise, cash design is available for other purposes.

Historically, additional depictions added value for the issuing authority. Coins showed the portraits of rulers (e.g. Maria Theresia's portrait on a one Kreutzer coin[1]), religious symbols (e.g. Austrian five Schilling coin depicting the crowned Madonna with nimbus and the infant Jesus standing on clouds[2]), propaganda (e.g. a Sesterce shows Nero giving a speech in front of soldiers[3]) and advertising (e.g. an Austrian 50 ATS coin advertising 50 years Österreichischer Rundfunk (Austrian Broadcasting) and parabolic dish antennas[4]).

The mass media have become a more effective means of advertising than images and slogans on the currencies can. Images on coins are no longer needed to advertise the head of state. Heads of state change more frequently in democracies and it is no longer practical to change all circulation coins with each change in government[5]. These developments create a vacuum of *usefulness* for the *issuing authority*, which now opts for noncontroversial designs. Weeds, critters, gestures, windows and fictive bridges are currently fashionable, but they lack purpose and the space is available for useful applications. Cash design can add value for its *users*. Measurement functionality is such a useful addition.

Banknotes and coins are essential for trade as a legal tender for the payment of goods and services. In the same context, metrology objectifies properties of goods. Some goods are sold in quantities of units of measurement, such as food per gram. Measurements provide a way to verify seller claims and prevent fraud. The union of currency and metrology will combine objects that are already individually valuable for the trade of goods. Accordingly, those measuring devices that facilitate trade should preferably be implemented as cash. In particular, circulation currency should be minted with units of measurement for size and mass because these are important for trade in food.

Artner described how coins are already used as a size reference[6] and size measurement[7]. Cash with measurement features provides the general public with measurement devices that they carry readily available in their purses for everyday use[8].

Contribution

Circulation currency is augmented with measurement functionality. Banknotes are augmented with rulers and protractors. Coin prototypes with rulers and measuring wheels are tested. Coins that have their standardized mass written on them can be used as weights and to test electronic scales. Protractor designs are developed for opaque coins, transparent coins and for coins with holes. Fruits can be compared with color scales to determine ripeness and prevent ingestion of spoiled food. Finally, a special system of rectangular coins is developed. Each coin in the set has a useful length that is a multiple of one centimeter, each thickness is a multiple of one millimeter and each mass is a multiple of the gram. The coins in this set can be intuitively combined for measurements. A mathematical framework is developed, where coin properties are designed to

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[1] KHM MK 2743aα.
[2] KHM MK 194909.
[3] KHM RÖ 5562.
[4] KHM MK 213400 and BGBl. 1974.
[5] For example, Austria has had nine different chancellors in the last ten years.
[6] Artner 2024a.
[7] Artner 2020.
[8] Artner 2024b.

be linear (homogeneous and additive) to allow their intuitive combination in measurement applications. The coins are combinable linearly for length, height, area, volume, mass, and monetary value measurements with reduced measurement error. Prototypes are built of each design and use cases are shown. The design process is presented in Sec. II Measurement errors of current coin shapes are discussed in Sec. III. Reference designs for existing cash shapes are presented in Sec. IV. Finally, a linear system of coins is presented in Sec. V where these measurement errors are reduced. Prototypes and use-cases are developed for all designs to show feasibility[9].

Design Process
General design considerations:

- Remove decorative symbols, inscriptions, and advertisement from cash.
- Use the freed space for practical features that are of benefit to the cash users—preferably ones that supplement the core purpose of cash.
- Measurement tools can be desired features of cash when they facilitate the trade of goods.
- All properties of measurement currency correspond to the international system of units (SI)[10], or integer fractions or multiples of SI units.
- An issued set contains a unit: 1 g, 1 cm width, 1 mm height, etc.
- Label properties: "Mass: 2 g", "Length: 1 cm", and so on.
- Indicate the expanded measurement uncertainty.
- Technical devices that work with coins (money counters, beverage dispensers) shall check coins to see if they meet the specified tolerances and sort them out if they don't (as already happens with damaged and counterfeit cash).
- People might not expect to find measurement tools on currency. The markings shall be visible enough to be immediately noticeable, but not to an extent that they obscure important features such as face value, currency, issuing authority, anti-forgery features or usability features for the blind.
- People have difficulty quickly identifying coins where the size and the face values in a set are not ordered, as described by Fitousi[11] and Hasegawa[12]. If several coins in the set measure the same physical quantity, then the measurement units and face values shall be in the same order. For example, if it measures size then the smallest coin shall have the smallest face value, the largest coin shall have the largest face value and so on.
- Continuously improve cash with technical progress from other measurement equipment. For example: Coins are rigid and therefore similar to rulers. Design considerations for rulers shall be applied to coins.

People already use cash for measurement purposes. The designer can improve money to make such measurements easier and more precise. Size comparisons in photos are the most common[13]. The depth information of Lithuanian 2 cent and U.S. quarter dollar coins were used as measure for the accuracy of imaging systems' depth information[14]. The standardized mass and area of coins have been used as a unit for pressure "A hybrid membrane suspended on a tube edge is mechanically strong enough to support five coins (∼ 16.0 g) without rupture (Fig. **3D**)."[15]. Automotive enthusiasts measure tire thread depth[16] and balance a coin on their car's engine block and rev up the engine to demonstrate that the motor block's vibration is too small to topple the coin[17].

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[9] Leibniz 1684: A posteriori, however, we recognize the possibility of a thing, if we have the experience that the thing really exists or has existed; for what really exists or has existed is possible in any case.
[10] BIPM 2019.
[11] Fitousi 2010.
[12] Hasegawa 2020.
[13] Artner 2020; Artner 2024a.
[14] Vlahacos *et al.* 1998; Lisauskas 2014.
[15] Yang 2019.
[16] Bridgestone 2022.
[17] For examples see the Youtube videos Greenman Solar 2018; Watson 2021.

The state-of-the-art measurement features are more decorative than functional. Bullion coins typically state their mass; common denominations are in troy ounces (e.g. Krugerrand, Vienna Philharmonic), but bullion coins quantized in SI units are already available (e.g. Chinese Gold Panda). Factions and multiples of troy ounces and grams are already widely used on bullion coins. The Vienna Philharmonic is available in 1, 1/2, 1/4, and 1/10 troy ounce denominations and the Gold Panda is available in 1, 3, 8, 15 and 30 g denominations. Bullion coins can't be brought into circulation for measurement applications, because their material value makes them prone to shaving and clipping, and people hoard them and trade with fiat money instead, which Aristophanes already described[18]. Some coins feature protractors, for example the 25 Euro *Big Data*, 20 Euro *The Dream of Flight* and 2 New Zealand dollar *Terra 2023* coins. Clocks, that are hidden inside coins, are called coin watches, but they might not be practical inside circulation coins due to their cost and fragility. We can view coin watches exemplary for many other mechanisms and electronics that could be built into coins, but are too complex for circulation, because they can't be mass produced with minting processes. Further, a patent search reveals all kinds of coin-tools such as wrenches, protractors, rulers, spark plug gappers[19].
We can't naively assign monetary value to any existing tools and print a face value on any device. The measurement functionality needs to be useful to a wider audience, for example spark plug gap tools[20] are already similar to coins, but they are not useful in everyday situations. There are also practical constraints on material, size, and shape. Coins are almost exclusively made of metal. Diameters of about 15 mm are the lower practical limit for modern circulation coins. Examples are 1 Belarusian Kopek (15 mm), 10 Mexican Centavos (14 mm) and 1 Spanish Peseta (14 mm). A diameter of 31.8 mm is required for a 10 cm circumference for applications as a measuring wheel. 30 mm diameter is also the practical size limit: USA Half Dollar (30.61 mm), 5 Swiss Francs (31.45 mm), and 50 Australian Cents (31.51 mm). Heights between 1 and 3 mm are typical: Canadian 10 cent to Hong Kong 10 dollar.

Mass ranges from about 1 g to 15 g. Examples are historic Austrian 2 Groschen (0.9 g)[21], Japanese Yen (1 g), 10 Malawi Kwacha (15.1 g), and Australian 50 cent (15.55 g). I will later suggest that coins in a series should span masses from 1 to 10 g and such values are practical. Coins are commonly round with rare exceptions such as regular polygons and scalloped edges, for example the historic Israeli 0.01 pound, and the 2 Hong Kong dollar. Rectangular shape examples are found in historic Japan and the Aruban Florin 50 cents. Square coins have two distinct disadvantages–they do not roll and their diameter is not uniform. This can make recognition and sorting in vending machines difficult. Colors are currently rare on circulating coins and common on banknotes. The commemorative circulating Canadian 25 cent show that colored coins withstand the wear of daily payment transactions. Size and mass abrasion from wear and tear remain reasonable for measurement applications even after decades of use[22].

Accuracy and Measurement Error Considerations

The measurement capabilities need to be intuitive. Pieces should be combinable in a meaningful way. In this section, sources of measurement uncertainty on today's currencies are discussed. We use them later to design cash with improved measurement features. Mathematically, we will demand that combinations of coins for measurements are linear mappings in the sense of linear algebra. In order to be considered linear, mappings need to be homogeneous $f(c\mathbf{a}) = cf(\mathbf{a})$ and additive $f(\mathbf{a} + \mathbf{b}) = f(\mathbf{a}) + f(\mathbf{b})$, where a, b are vectors and c is a scalar. Without going into further details on linear algebra, we will interpret coin features as vectors and the function f as a measurement of a measurand.

This may sound complicated, but the nominal values of modern fiat coins already work in this mathematical way. The combined value of five two Euro coins is 5×2 EUR = 10 EUR and they can be exchanged for a 10 EUR banknote of equal value. A penny and a quarter have a

[18] Eliot 2014.
[19] V. S. Voronov patented several coin tools, the most relevant for this work are probably Voronov 2001 and Voronov 2002.
[20] Martin 1977.
[21] BGBl. 1950.
[22] For examples see Cope 1969; Velde 2013; Manas/Velde 2021.

combined value of 0.01 USD+0.25U SD = 0.26 USD. This wasn't possible in currencies without face value, such as silver and gold bullion. Multiple silver coins didn't add to the value of a gold coin unless there was a definition by law. The value of a silver coin and a gold coin couldn't be added in a meaningful way.

Measurement problems that might arise with circular coins are shown in Fig. **1**. Fig. **1a** shows coins that are intended to be used as a measure for length and aligned along their centers to do so. During the alignment of the coins, it might happen that their centers are not perfectly aligned. The mapping is then not homogeneous as the length measured with three identical coins a isn't the same as three times the length of coin a, or length(3a) ≠ 3 length(a). If we use identical coins (or coins that have the same diameter), then we could use a straight bar to align their centers, but this isn't possible for coins with different diameters as is shown in Fig. **1b**. The example illustrates that such misalignment is not additive as the length measured with coin a and b isn't the same as the length of coin a plus the length of coin b, or length(a + b) ≠ length(a) + length(b). A measurement error for height is shown in Fig. **1c** where coins of different sizes are stacked to measure height, but the larger coin's edge is thicker than its faces. The resulting measurement is not additive as height(a + b) ≠ height(a) + height(b). In a mathematical sense, such misalignments are no longer linear mappings. The given examples are exaggerated and a user would likely not notice the deviation, which then results in measurement errors. We could demand that people only use coins in the proper ways, but this would be illusory of us. Coins need to be designed such that people can use them for measurements, intuitively combine coins in their measurements and that the coin design itself aims to minimize measurement errors.

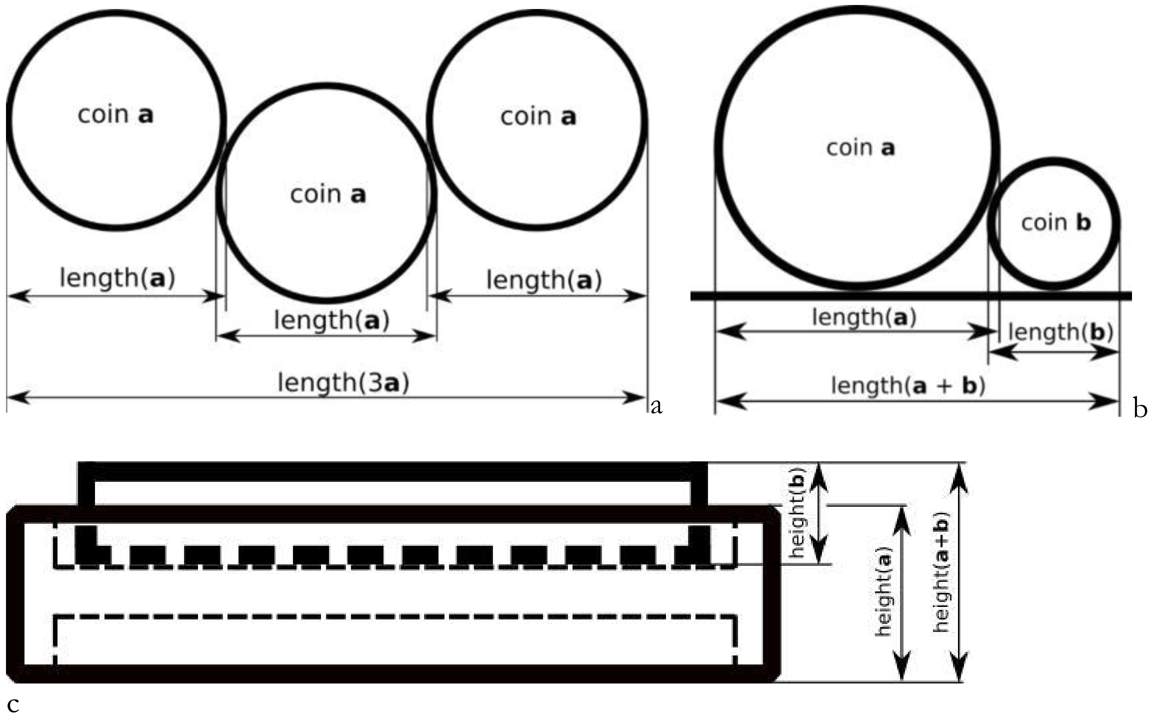


Fig. 1. Practical problems that result in errors when measuring length with circular coins. The problems are exaggerated for illustration purposes. a) Misaligned centers result in a measurement that is not homogeneous, i.e. length(3a) ≠ 3 length(a), b) Aligning the coins along a straight surface could counteract the misaligned centers, but doing so is not additive with coins of different diameter, i.e. length(a+b) ≠ length(a) + length(b). c) Error in height measurements with different sized coins. An edge that is thicker than the faces results in a measurement that is not additive, i.e. height(a + b) ≠ height(a) + height(b).

For protractors, consider the 2 New Zealand dollar coin *Terra* from *Pressburg Mint*. We need to align the coin center at the vertex. However, the coin is opaque which makes alignment difficult and subsequently creates in measurement errors as shown in Fig. **2a**. Skilled mathematicians might of course apply advanced techniques. For example, they could extend the straight lines and aim to align the coin center with their intersection.

They will surely be aware of Thales' law of vertically opposite angles and align the coin such that its protractor reads α = β, for which they need arithmetic operations on the numbers shown on the protractor. Good design should never require advanced calculations. Further, the protractor markings don't extend inwards towards the center and they don't extend outwards along the rim which makes precise readings of objects smaller or larger than the scale difficult.

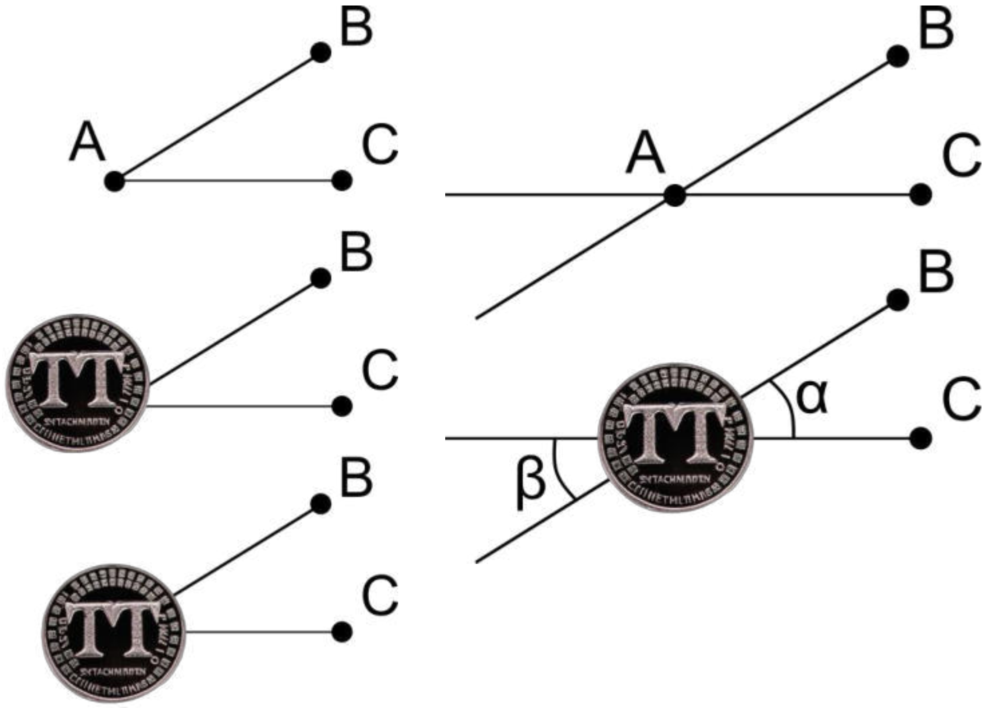


Fig. 2. Practical problems when using opaque coins with protractor designs. The coin design is AI generated[23]. a) It is difficult to properly align the center of opaque coins. Misalignment results in measurement errors when determining the angle. b) The coin could be properly aligned by extending the straight lines, knowledge of Thales' law of vertically opposite angles and further calculations done on the numbers displayed on the protractor. However, good design should not rely on the user being knowledgeable in such advanced techniques.

Reference Designs

Several reference designs are proposed in this section that augment cash with measurement functionality. Some can be added to existing cash; others introduce special changes to size, mass and form.

Banknotes with rulers - measuring tape

Banknotes are flexible, but not elastic. Banknotes are well suited to become rulers and measuring tapes. Euro notes already have evenly spaced lines along their left and right edges, but they are not useful. Fig. **3** shows how they can be augmented with rulers without additional manufacturing cost. The ruler is unimposing on the general design.

[23] Artner 2022.

a

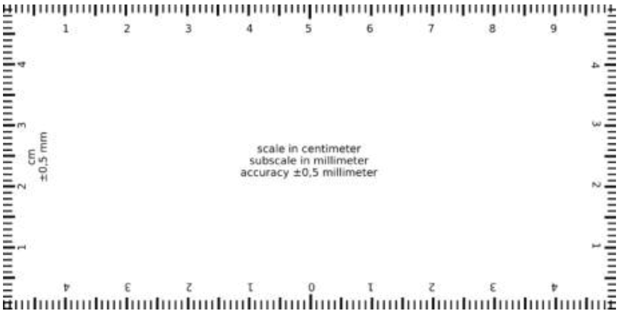
b

c

Fig. 3. Banknotes are augmented with rulers. a) 50 Euro prototype with a ruler along the top edge. b) A banknote measures the length of a protruding pipe. c) A prototype is used as measuring tape to determine the circumference of a metal rod.

Banknote size aligned with SI units

The next step is to adjust banknote length and width with the SI. The dimensions of the banknote in Fig. **4** are aligned with SI units, in the example 10 cm x 5 cm. The whole bill becomes a measuring device and simplifies measuring lengths that go beyond a single note. The user simply marks the end of the note (in the simplest form with a finger) and replaces the note's zero on the marking. They do this until they reach the end of the measured length. The total length is 10 cm times the number of times they've placed the note + the value read from the ruler. Doing the multiplication as mental arithmetic is a lot easier when the banknote is 100 mm x 50 mm then for example with a 100 Euro that is 147 mm x 82 mm.



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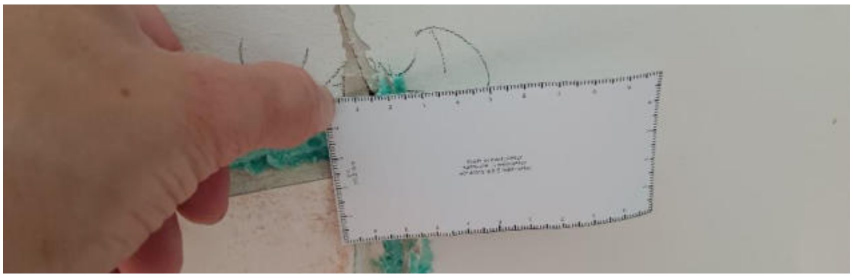
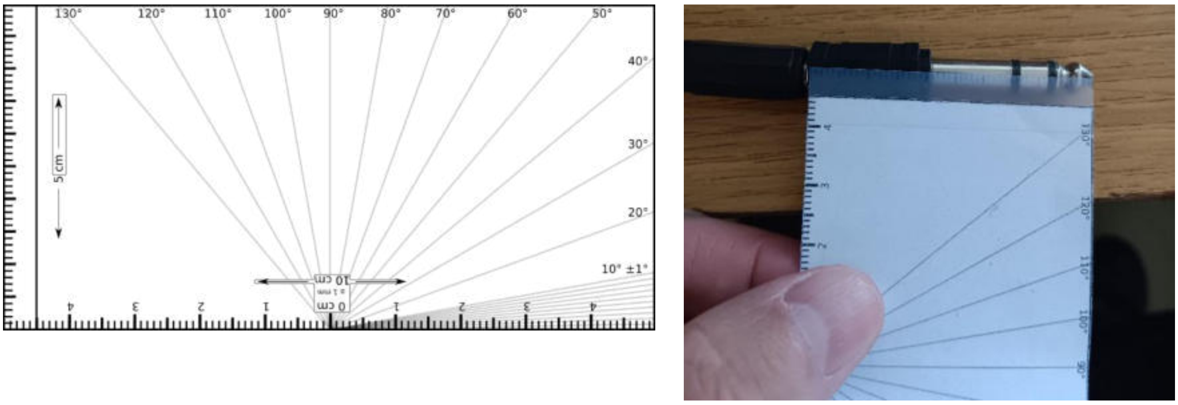

d

Fig. 4. Banknote design with rulers as frame. The size of the note is 10 cm x 5 cm, the rulers have centimer and millimeter subdivisions. a) Drawing and b) Measurement of a provisionally repaired wall section. Align banknote zero as usual, but measured length goes beyond note. Mark end of note with pencil or finger. c) Shift zero to marking. Repeat until you reach the end of the measured length. Count how often you've placed note. d) Read remaining length from ruler. Total length is 10 cm times the number of times you've placed the note + the value read from the ruler at the end.

Banknotes with transparent windows

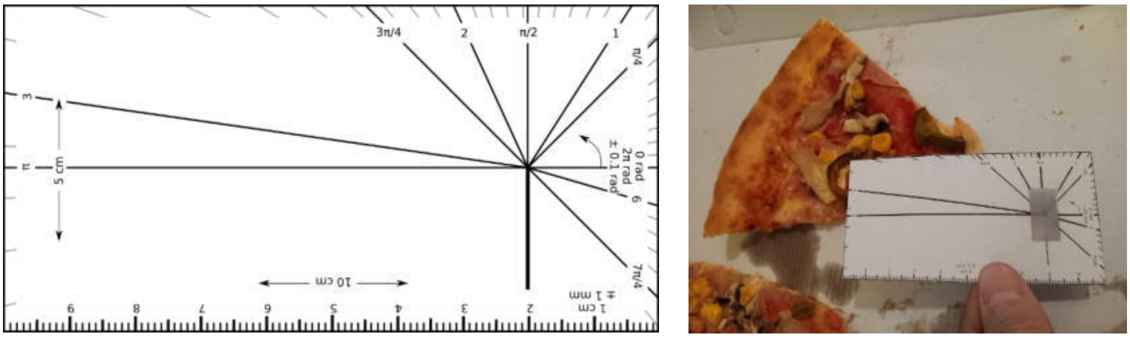
Modern banknotes have transparent windows as an anti-forgery feature[24], for example Canadian dollar, Euro (50 and above) and the British pound. The protractor and ruler centers can be placed in such a window to help with alignment. Fig. **6** shows a reference design and a prototype of a banknote with a transparent window. The protractor center is positioned in the window to assist the user with alignment.

[24] Singh 2008.



a b

Fig. 5. Reference banknote design with SI unit dimensions (5cm × 10cm). The left and lower sides feature rulers, the ruler on the left is transparent. The upper and right sides are protractors with angles denoted in degrees. One-degree steps are shown in the lower right corner. a) Drawing and b) Prototype with transparent ruler is used to measure the length of an audio jack.



a b

Fig. 6. Banknote designs with a transparent window (5cm × 10cm). The transparent window shows the protractor center and important angles to make aligning easier. a) Drawing and b) Prototype, laser-printed and marker on transparent tape. The lines in the bill's center are thicker to be more noticable while the lines in the window are thinner to be more precise. It is used to measure the angle of a slice of pizza.

Coins with Mass, Diameter and Height

The first step towards adapting measurement coins is to simply state their mass, diameter, height, volume on the coin, an example is shown in Fig. **7a**. Use case: Testing of scales to prevent fraud. Consumers have reference masses readily available in their pocket and can use them to test scales. They simply take a coin out of their wallet, place it on the scales and compare if the number on the coin matches the amount shown on the scales, see Fig. **7c**. Under Austrian law, the scales are subject to official verification. The device has an orange sticker on it that shows the Austrian coat of arms, the Eichstelle by number and the validity year. However, while rigorous laws and procedure prevent tempering, the consumers only see a sticker. The proposed coin design provides consumers with an easy-to-use measurement device that they carry in their purse and that they trust. The coins facilitate trade not only through their monetary value, but by reducing mistrust between buyer and seller.

Use case: Size comparison. People use coins for size comparison in photographs[25]. They can continue to do so and additionally the coin states its dimensions for viewers who are not familiar with the shown coin or currency.

[25] Artner 2020; Artner 2024a;Artner 2024b.

a

b

c

Fig. 7. Simple augmentations that can be added on any existing coins. a) Prototypes of 1 and 2 Euro coins that are labeled with mass and diameter. b) Use case: Coins for size comparison. The size comparison works as usual, and the inscription is useful for people who are not familiar with the currency. c) Use case: Testing of scales to prevent fraud. A coin is placed on a scale in the produce section of a supermarket to validate it. The consumer compares the value on the scale to the value on the coin.

Coins with Rulers

A simple ruler can replace flowers and busts or supplement them. The *Riga Technical University* 1 Lats collector coin[26] contains both a rudimentary ruler and protractor. One design was created with the help of artificial intelligence[27]. Like in a previous work on AI generated coin depictions[28] DALL-E 2[29] was used for inspiration, see Fig. **8a**. Based on the AI draft, a reference design is developed that can be retrofit onto a 2 Euro coin. It is depicted in Fig. **8b**. The ruler shows both inches with fractions of inches and centimeters with millimeter subdivisions. Such dual-ruler coins might be useful to introduce a population to the metric system that is not yet familiar with SI units for length. The ruler is aligned on the coin such that the coin width at the ruler's position is a somewhat round number, this particular design places the rulers at 2 cm and 1 inch width.

Prototypes were made from stainless steel and are shown in Fig. **8c**. The coarser example of the left was manufacturer with engraving spray while the finer lines on the prototype on the right were achieved with fiber laser engraving. Instead of holding the coin to the object that is being measured, users hold the object onto the coin. This is shown in Fig. **8d** in which the length of cloves is measured.

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[26] The coin consists of two triangles. One of them contains a ruler and the other a protractor. However, the lines are not labelled and the design is not functional due to a stub in the center.
[27] AI generated coin designs are also described in Machado 2024.
[28] Artner 2022.
[29] Ramesh *et al.* 2020, available online as OpenAI DALL-E 2, https://openai.com/de-DE/index/dall-e-2/.

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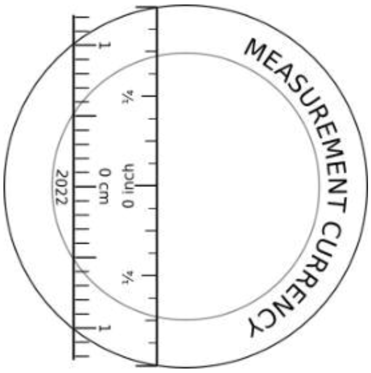

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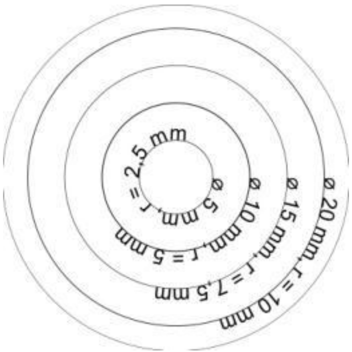
Fig. 8. A reference design with a ruler on the coin's face was developed with the help of text-to-image AI. a) Original image created with DALL-E 2, prompt "a coin with measurement features," b) Reference design, c) two prototypes and d) used to measure the length of cloves.

Circular Rulers

Coins are mostly circles, so an adapted ruler design would be beneficial to retrofit existing coins. A coin design might have a ruler as shown in Fig. **9** with concentric circles that form a ruler in all directions. The design was inspired by the concentric circles of Lichtenberg figures (Concentrische Circkel)[30]. This would also be beneficial for measuring the diameter of circular objects such as rods and tubes as they can be centered visually on opaque coins. However, too many circles would clutter the appearance and would be more confusing than helpful. The basic design is also prone to measurement errors as the exaggerated example with a screw in Fig. **9c** shows. Radial rulers are added to prevent such errors and allow more precise measurements. Finally, a protractor can be added along the rim. This is demonstrated on the example of a button in Fig. **9f**.



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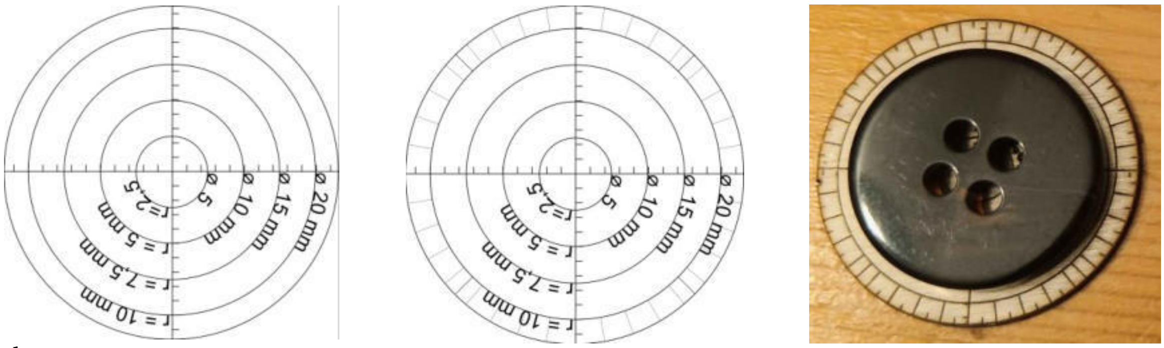
[30] Geometric shapes that are produced by electric discharge that were discovered by Georg Christoph Lichtenberg.



d e f

Fig. 9. a) Proposed coin design that has a ruler with 2.5mm increments (or 5mm increments on the diameter) where the ruler divisions are concentric circles. b) Measurement of pierres de strass (rhinestone) on a paper prototype. c) The design can also be used as a ruler, but measurement errors occur when objects are not aligned radially. The screw is actually 9.3 mm long, not 7.5 mm as the erroneous use might suggest. d) Improved design with ruler and millimeter subdivisions. e) We might add a protractor to the edge at a later time, once the population gets used to measurement features on coins and we can omit the labels. f) Measurement of a button. The concentric circles make it easier for users to visually center opaque circular objects. Subdivision increases measurement precision, but simply adding more circles would clutter the design and reduce readability.

Transparent Coins

The problem of aligning the coin center in Fig. **2** can be solved with transparent coins. Lines and text are placed on the underside, to avoid parallax and make alignment easier. Coin designs can be derived from transparent protractors or geometry triangles[31]. The prototypes in Fig. **10** were drawn with Staedtler permanent marker on transparent bingo chips.

a b

Fig. 10. Transparent coins a) Ruler, b) Protractor used for markings in origami and paper-craft.

Protractor Coins with Holes

Several currencies have coins with a hole in the center, perhaps most famously the circular Chinese coins with square holes (方孔钱) that were in use from the 4th century BC until the 20th century AD. Fig. **11** shows a simple protractor design for coins with holes. The design provides markings in 5° steps and emphasizes directions that are multiples of 45°. Only lines in 10° and 45° increments extend towards the center because of the small size. Prototypes were produced by fiber laser engraving and laser cutting and are the same size as one Danish Krone (1 DKK) coins as shown in Fig. **11b**. In Fig. **11c** the medal is used to check the angles in a technical drawing[32]. The design is labeled with ± 1° uncertainty, however this might increase in practice as the example demonstrates. The lines help with alignment of the vertex in the protractor center, but there is still small misalignment even with reasonable effort.

Fig. **11d** shows a bi-material variant with a crosshair on a transparent window. The crosshair lines are drawn on the underside of the window, such that the coin can be aligned and the values read correctly when it is viewed from above. Note how the coin is held slightly tilted and the lines appear to be slightly off center due to parallax. The lines should extend along the

[31] Zauner 1956; Bradler 1956.

[32] Artner *et al.* 2019.

edge (both outer and inner) to aid alignment. Transparent centers can be achieved with novel bi-material coins as described by Afonso *et al.* [33]. For Chinese coins the hole corners might preferably coincide with angles that are either multiples of 45° □ or 90° ◊.

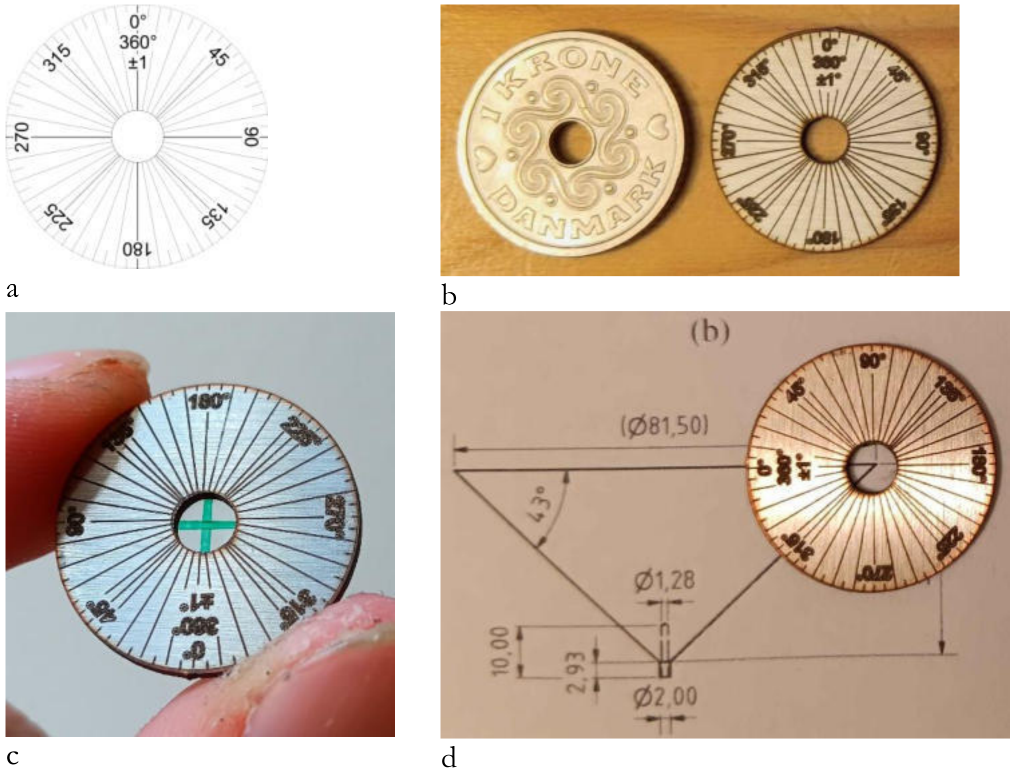


Fig. 11. A protractor coin with a hole in the center. a) Reference design, b) Laser engraved prototype next to a one Danish Krone (1 DKK) coin, c) Variant with a crosshair on a transparent window. d) Use case, the markings around the hole make alignment easier, although not as precise as transparent coins.

Opaque Protractor Coins

We recall from Fig. **2** that it is difficult to align opaque coins. We could imagine a semicircle shape that many protractors use, but a semicircle is more prone to bending than circular coins and sharp corners might be an issue in everyday wear and tear (both on the coins and on purses). A design is proposed in Fig. **12** where the protractor is aligned to the coin edge. The first coin design was inspired by the flag of Seychelles[34]. Fig. **12a** shows the design process and a reference design for protractor coins. Fig. **12b** shows a prototype of a colorized coin as it might be used by the Seychelles. The prototype is made as casted metal with soft enamel. A combination with national symbols might be an intuitive way to introduce the public to protractor designs on coins.

A reference design is given in Fig. **12c** with the reference point at the coin edge. Important angles are shown as lines (here 0° and ±45°), angles are marked in 5° steps and labeled in 10° steps. 5° steps should be practical on circulation coins, denser steps are technically possible, but are difficult to distinguish on such small disks. Note that the proposed design has a limited measurement range. It is well suited to measure acute angels, but obtuse angles require calculations by the user and might be difficult to read from the scale. Reflex angles also can't be measured directly by the user and require measurements of the corresponding angles and calculations.

[33] Afonso *et al.* 2019.
[34] Seychelles 1996.

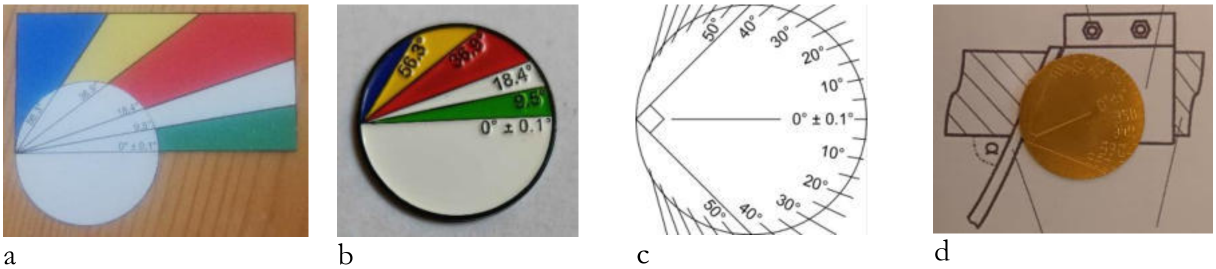


Fig. 12. A protractor coin with the reference point on one side inspired by the flag of Seychelles. a) An important use case is of course to check if the angles on a flag of Seychelles are proper and b) Colored prototype in the size of a 1 Seychelles Rupee coin (diameter 25.5 mm). c) A protractor coin with the reference point on one side. d) The protractor coin is used to measure and angle in a technical drawing[35].

Colored Coins as Measurement Tools

The Canadian quarter dollar coins demonstrated that colored circulation coins are feasible. Colors are proposed as measurement tool on cash and are designed to augment trade. Fig. **13** proposes a design that promotes produce: Avocado fruits. The product pictogram provides context to a color scale. Users will intuitively deduce that the color scale measures ripeness of avocados. Fig. **13b** shows the use-case: The medal is held such that the color scale is close to the fruit. The user compares the fruit's color to the colors on the scale and reads the description: The fruit is ripe. The prototype was built as a collage with laser-printed paper glued onto an aluminum blank. The avocado sketch was generated with DALL-E 2. Fig. **13c** shows a variant with a banana ripeness chart.

Fig. 13. A prototype that uses colors to supplement trade of foods. The medal promotes a fruit and provides a colored scale measures ripeness. a) Reference design with avocado ripeness chart. b) Use case: The medal's color scale is held to the avocado fruit. The scale color is matched with the fruit color. The user reads the text on the matched color. The fruit is ripe. c) Banana ripeness coin.

Measuring Wheel

These devices are known under several names such as curve runner, rolling ruler, rolling measuring tape and larger versions as measuring wheel. Measuring wheels are popular for measurements of long distances that are not necessarily straight lines. Coins can be built similar

[35] Artner 2021.

to rotary line measures[36], but a grip or holder in the center is likely unpractical for currency. Fig. **14a** replicates the figure of an 1887 patent. Without a physical grip, we might design the coin such that users can use everyday objects like pens as axle, see Fig. **14b**. People use their fingers as axle to roll and steer the coin; no other objects are needed.

Fig. **14c** shows prototypes made from copper, brass and stainless steel with different hole diameters. The coin should be engineered with a diameter of $d = \frac{10\text{cm}}{\pi} \approx 3.18\text{cm}$ to obtain a circumference of 10cm. I've approximated the prototypes with 32mm diameter blanks, because these are inexpensive and available in bulk. The holes were turned on a lathe and the markings were drawn with permanent markers. Users roll the coin along the path as shown in Figs. 14d, 14e and 14f. They apply a downward force to increase friction and avoid gliding (compare Amontons' laws of friction). The edge can be reeded to increase the coefficient of friction. Different hole sizes will certainly be preferable to different people. Further research is needed to find a good fit of the hole and reeded edges for the public.

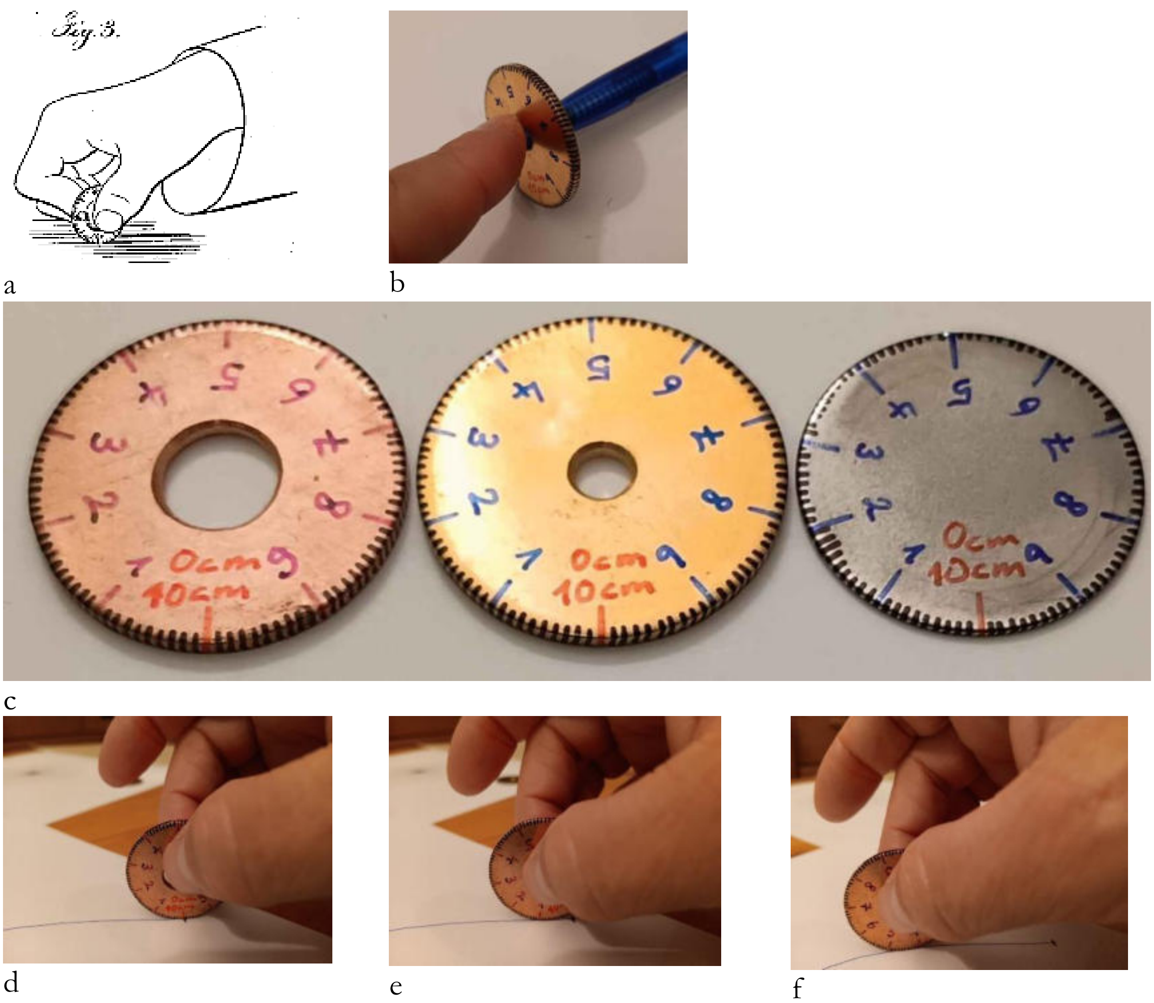


Fig. 14. Coins can be made into measuring wheels. a) Depiction on how to hold the 1887 patent of A. Stoner on a grip in the center. b) Prototype used with a pen as axle. c) Prototypes with different hole diameters were tested. Permanent marker on metal blanks. d to f) Instructions: Users set the coin with the "0 cm" marking onto the starting point. Then they push the coin forward along the line. When measuring long distances, users need to remember how often the "0 cm/10 cm" marking has passed the ground.

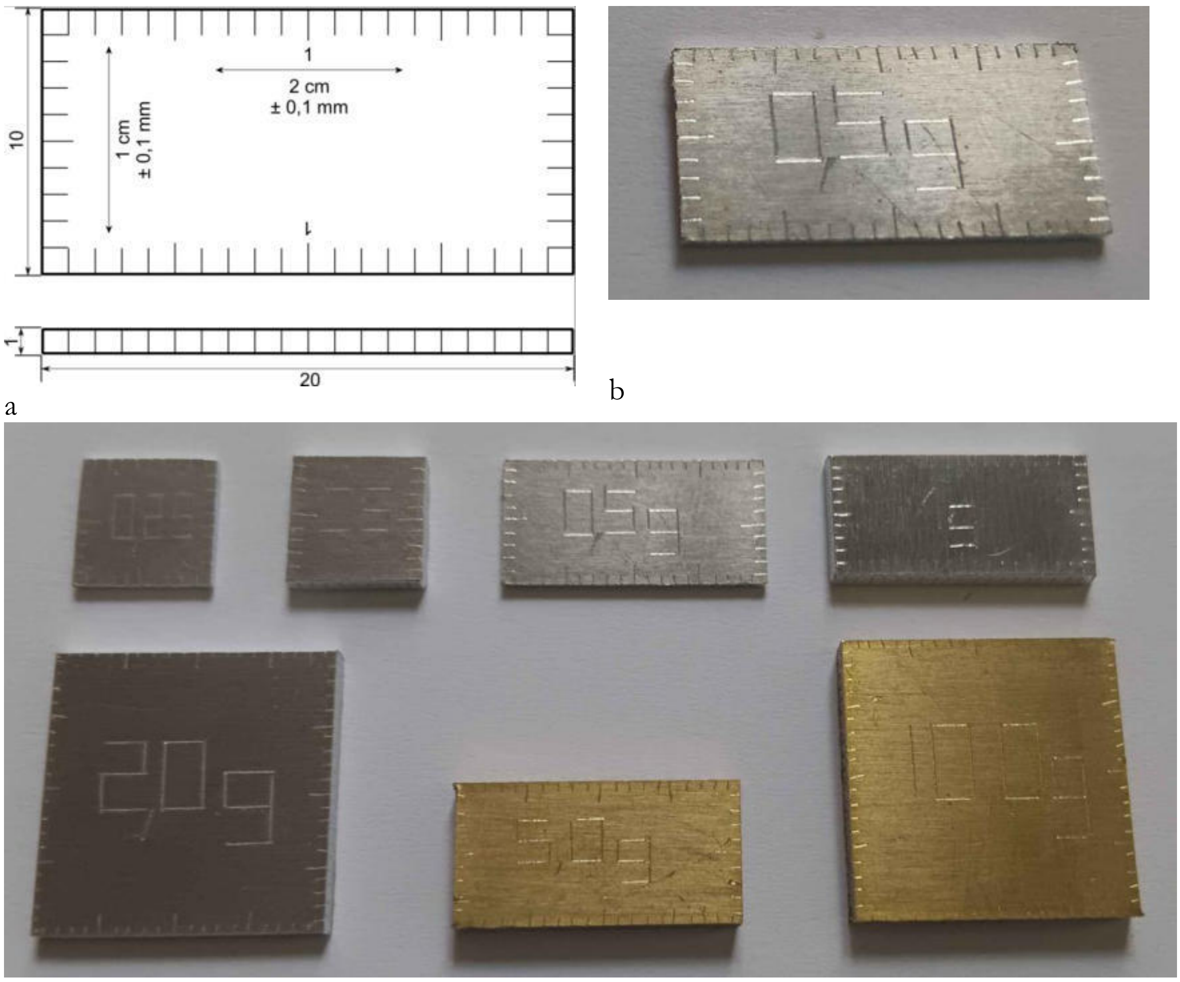
A Linear System of Coins

We have demanded in Sec. II-A that measurement devices shall combine into a linear measurement system and that they contain at least one device with unit value for each quantity. In this section, a system of coins is presented that can be used to measure length, width, height, area, volume, mass (weight), and monetary value.

[36] Stoner 1887.

We recognize that typical coins have a diameter in the centimeter region, a height in the millimeter region, and a weight in the gram region. Fig. **15** shows the proposed reference design. The design has a unit width of 1 cm and a unit height of 1 mm. A rectangular shape is used, because several coins can be strung together and the measurement will remain linear. Users can simply reduce measurement error by pushing the coins together. The design is flat without rims at the edge so that stacking coins is also a linear measurement system. The linearity of the proposed design is demonstrated in Fig. **16**. Notice how the coins only need to be aligned in the desired direction–by simply stacking them or pushing them together. Length measurements are homogeneous length(3a) = 3 length(a) and additive length(a + b) = length(a) + length(b). Height measurements are homogeneous height(2a) = 2 height(a) and additive height(a + b) = height(a) + height(b). Misalignment in another direction does not cause measurement errors as it did in Figs. 1 and 1c. The face values of modern fiat currencies can already be added intuitively in a linear way.

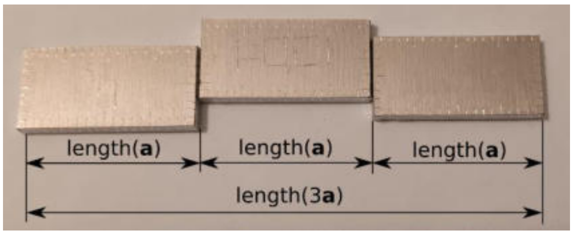
Now that length, width, height and rectangular shape are fixed, the mass is controlled through the material density. The prototypes are achieved with standard aluminum $AlMg_3$ and brass. The edges can be rounded for practical reasons as sharp corners are prone to wear and might cause damage. Ideally, we would only round the outermost millimeter to preserve as much of the rulers as possible. Fig. **17** shows that the masses of the proposed medals are linear. The medals can simply be stacked or piled up to add their mass.



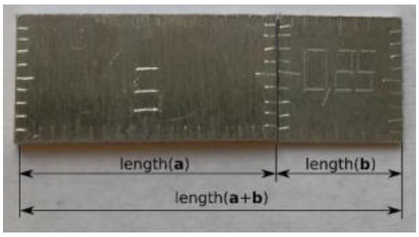
c

Fig. 15. A system of rectangular coin designs with unit width of 1 cm, unit height of 1 mm and unit mass of 1 g. a) Reference design with 10 mm x 20 mm x 1 mm, b) Prototype closeup. c) A compatible system of coins with size and mass measurement functionality. The sides are multiples of 1 cm and the thicknesses are multiples of 1 mm, the masses are fractions and multiples of 1 g. The prototype medals are labeled with their respective masses and their sides are rulers with millimeter subdivisions. Length and mass scales each span over a decade throughout the set. The individual pieces in the set can be combined intuitively as a linear measurement system. From top left to bottom right: 10x10x1 mm with 0.25 g, 10x10x2 mm with 0.50 g, 10x20x1 mm with 0.50 g, 10x20x2 mm

with 1.00 g, 20x20x2 mm with 2.00 g, 10x20x3 mm with 5.00 g, 20x20x3 mm with 10,00 g (mass precision ±0.03 g).



a



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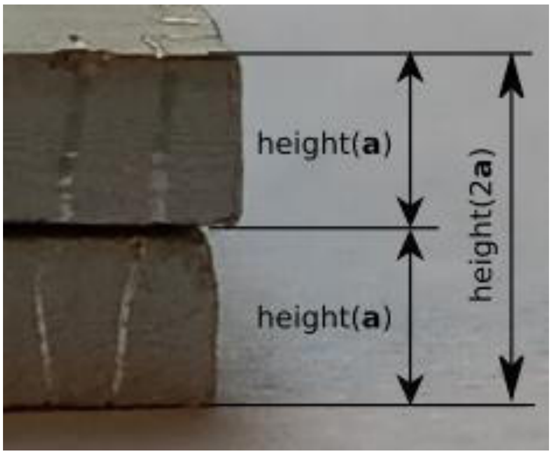

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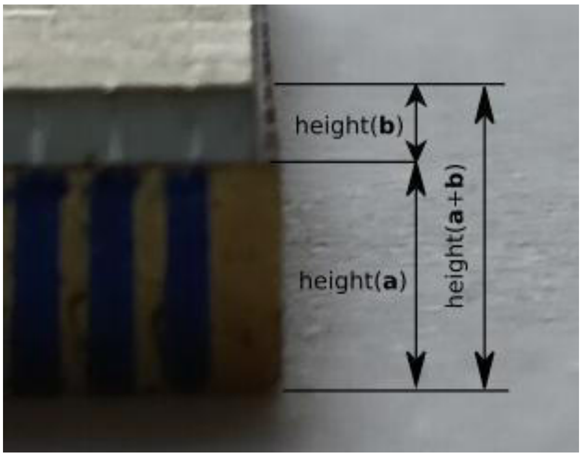


d

Fig. 16. With the proposed coin designs, length and height measurements are linear mappings in the sense of linear algebra. This allows them to be intuitively used as measurement tools. The coins can simply be pushed together for length measurements and they can be stacked for height (thickness) measurements. a) Length measurements are homogeneous, b) Length measurements are additive, c) Height measurements are homogeneous, d) Height measurements are additive.

a

b

Fig. 17. The masses are aligned with the SI. Additionally, the masses are linear and can be combined intuitively. The accuracy of the Leuchtturm Libra 100 scales is ± 0.03 g. a) Mass combinations are homogeneous. The masses of three 1 g medals are measured as 3.00 ± 0.03 g. b) Mass combinations are additive. The masses of one 10 g and one 0.5 g medals are measured as 10.50 ± 0.03 g.

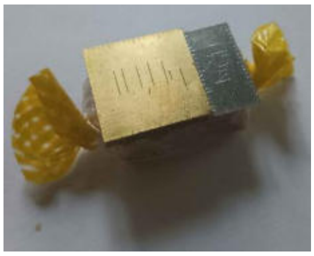
Considerations for the Design of a Series

From the proposed design, we can trivially derive smaller and larger designs with dimensions that are integer multiples of our base units, e.g. 2 cm × 2 cm coins, 3 cm side length, 2 mm thickness, or 2 g mass. We can use different materials to achieve higher mass with similar dimensions. It might be more reasonable to adapt the 1, 2, 5 scale of Euro coin monetary values also for mass, i.e. 0.5, 1, 2, 5, 10 g, or quarter, half, full values for countries with the imperial system. In practice, the choice of corner radius, motive depth, material alloy, and special geometries like bimetal coins should provide enough degrees of freedom to engineer the desired masses for desired sizes. I present prototypes in Fig. **15c** that form such a measurement system to demonstrate feasibility. The system contains pieces with 10x10 mm, 10x20 mm and 20x20 mm sizes, 1, 2 and 3 mm heights, and masses from 0.5 g to 10 g. A north American system might use halves and quarters instead of a 1-2-5 scale and a quarter gram piece is shown for reference.

Use case size measurement

The proposed system includes the benefits of the previous designs. The labeled face value on the reverse allows linear combinations of monetary value. Their use as size comparison is improved if we label the length, width and thickness. We expect that the public will get quickly used to coin sizes being aligned with the SI and then these labels can be omitted. Fig. **18** shows some examples with a piece of fudge. Coins can be pushed together and stacked. These combinations will intuitively also be aligned with the SI. People can read millimeter values off the rulers. The rectangular shape aligned with the SI means makes area and volume measurements easy.


a


b


c

Fig. 18. Two coins from the proposed linear system are combined to measure a piece of fudge that is larger than a single coin in the series. a) Two pieces are combined to measure length. b) Two pieces are combined to estimate area. c) Several pieces could be stacked to measure height, but a klippe doesn't roll and standing it upright next to the fudge is probably the simpler solution.

Use Case Roberval Balance

The proposed coins can be used as masses for weighing scales. Coin masses (weights) can be linearly combined. Fig. **19** shows mixed peppercorns being weighed on a Roberval balance with the proposed design. The combined mass of all coins placed on the same tray is the sum of their individual masses. Mistrust between buyer and seller is a well-known issue in trade. If a party suspects that a weight might be tempered, then they can test their coin on the balance. The coin design provides the general public with a set of reference masses that stay with the consumer. A merchant should have no objections to using a customer's coins as weights as the coin masses are then also overseen by the state.

Fig. 19. The mass of a pile of mixed peppercorns is weighed on a Roberval balance with the proposed coin design. The balance equates the mass of the peppercorns to the mass of the coins. The mass of the coins is determined by adding the individual masses on the coins as 10 g + 5 g + 2 g + 1 g = 18 g. Therefore, the mass of the peppercorn pile is 18 g.

Conclusions

I recommend cash with a practical benefit for the user, especially when they trade goods. Metrology is such a practical additional benefit because objectifies properties of goods that determine their value. Cash for metrological applications brings advantages in the trade of goods from a symbiosis with the already existing use of cash as a means of payment for goods. I have developed reference designs of coins with metrological functions and built prototypes. People carry cash for everyday use. Augmenting cash with measurement functionality provides the general population with simple metrological instruments that they have readily available.

Acknowledgements

Currency codes are given according to the ISO 4217 standard[37]. The author thanks Markus Greif from the Department of Numismatics and Monetary History at the University of Vienna for valuable feedback on the manuscript. The author thanks the Münzkabinett of Kunsthistorisches Museum Wien for digitizing several coins[38]. They are available in the interactive catalogue at www.ikmk.at. Numista.com is a great resource to search for coins. The prototypes of the DALL-E transition coins (Fig. **8**), circular rulers (Fig. **9f**) and protractors with holes (Fig. **11**) were manufactured by rausgebrannt e.U., Gumpendorferstraße 32, 1060 Vienna, Austria. The prototypes of the Seychellen coins in Fig. **12** were ordered from Pro Merchandise, Tijinkskamp 2a, 7631 AK, Ootmarsum, the Netherlands. The blanks for the mass-and-size system were cut by Lasercut e.U., Missindorfstrasse 21, 1140 Vienna, Austria. The blanks of the rolling measure prototypes in Fig. **14** were turned on a lathe by Christoph Pruckner Schlosserei und Schmiede, 3324 Euratsfeld, Lower Austria. The Leuchtturm coin scales in Fig. **17** were supplied by Vienna-Spezialitäten, Mariahilfer Straße 134, 1150 Vienna, Austria. The author thanks Heidi Hausmann for lending the Roberval balance in Fig. **19**. The help of Häupl & Ellmeyer patent attorney office, Mariahilfer Str. 50, 1070 Vienna, Austria, with the patent and design applications is gratefully acknowledged. I've drafted parts of the manuscript in my native language German and later translated them to English with DeepL[39].

[37] ISO 4217.
[38] KHM MK 194909, KHM MK 213400, KHM MK 2743aα und KHM RÖ 5562.
[39] DeepL, https://deepl.com.

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